Ve401 Probabilistic Methods in Engineering

Summer 2021 — Assigment 6

Date Due: 12:10 PM, Wednesday, the 30th of June 2021

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This assignment has a total of **(18 Marks)**.

Exercise 6.1 Symmetric Confidence Intervals Are Optimal

Consider a general two-sided $100(1-\alpha)\%$ confidence interval for the mean μ when σ is known:

$$\overline{x} - z_{\alpha_1}\sigma/\sqrt{n} \le \mu \le \overline{x} + z_{\alpha_2}\sigma/\sqrt{n}$$

where $\alpha_1 + \alpha_2 = \alpha$. Show that the length of the interval, $\sigma(z_{\alpha_1} + z_{\alpha_2})/\sqrt{n}$ is minimized when $\alpha_1 = \alpha_2 = \alpha/2$.
(2 Marks)

Exercise 6.2 Confidence Interval for the Standard Deviation

The percentage of titanium in an alloy used in aerospace castings is measured in 51 randomly selected parts. The sample standard deviation is $s = 0.37$. Construct a 95% two-sided confidence interval for σ.
(2 Marks)

Exercise 6.3 Confidence Intervals and Fisher Test for an Exponential Distribution

i) Suppose X follows an exponential distribution with parameter β. Show that $Y = 2\beta X$ follows an exponential distribution with parameter $1/2$.
(2 Marks)

ii) Verify that Y follows a chi-squared distribution with 2 degrees of freedom.
(1 Mark)

iii) Suppose $X_1, \dots, X_n$ is a random sample of size n from X. Show that

$$2\beta \sum_{i=1}^{n} X_i$$

follows a chi-squared distribution with $2n$ degrees of freedom.
(1 Mark)

iv) Use this information to find expressions for the confidence intervals for β, μ and σ^2.
(2 Marks)

v) Assuming that the data on Slide 292 of the lecture follows an exponential distribution with parameter β, find a confidence interval for β.
(2 Marks)

vi) Perform a Fisher test of the hypothesis $H_0\colon \beta \ge 0.01$ on this data. What is the P-value of the test?
(2 Marks)

Exercise 6.4 Confidence Interval for the Discrete Uniform Distribution

Let $X_1, \dots, X_n$ be a random sample of size n from a discrete uniform random variable $X\colon S \to \{1, \dots, N\}$. Let

$$X_{\max} = \max_{1\le i\le n} X_i.$$

Show that a $100(1-\alpha)\%$ "confidence interval" for N is given by the discrete set

$$\{X_{\max}, X_{\max}+1, \dots, X^*\}$$

where X^* is the largest integer such that $X^* \le \sqrt[n]{1/\alpha}\,X_{\max}$.
(4 Marks)