Ve401 Probabilistic Methods in Engineering

Summer 2021 — Assigment 8

Date Due: 12:10 PM, Wednesday, the 14th of July 2021

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This assignment has a total of **(30 Marks)**.

Exercise 8.1 Comparing Variances – Fischer Test

Prices for regular unleaded gasoline can vary widely from day to day and location to location. These data were obtained on June 1, 2001, from a sample of stations across the respective states (price is in dollars per gallon):

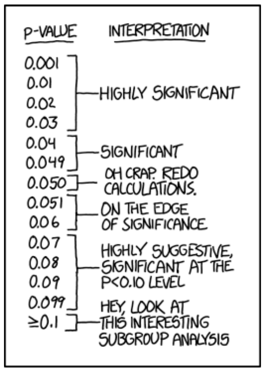
South Carolina

1.46	1.47	1.42	1.51	1.55
1.52	1.48	1.47	1.53	1.50

Michigan

1.69	1.79	1.72	1.76	1.80	1.91
1.59	1.89	1.72	1.63	1.55	1.71

Use these data to test for equality of variances. What is the P-value of your test, and what conclusions do you draw?
(4 Marks)



R. Munroe, *P-Values*, https://xkcd.com/1478/

Exercise 8.2 Comparing Variances – Neyman-Pearson Test

Two chemical companies can supply a raw material. The concentration of a particular element in this material is important. The mean concentration for both suppliers is the same, but we suspect that the variability in concentration may differ between the two companies. The standard deviation of concentration in a random sample of $n_1 = 10$ batches produced by company 1 is $s_1 = 4.7$ grams per liter, while for company 2, a random sample of $n_2 = 16$ batches yields $s_2 = 5.8$ grams per liter.

Is there sufficient evidence to conclude that the two population standard deviations differ by at least 10%? Use $\alpha = 5\%$. What is the power of the test?
(3 Marks)

Exercise 8.3 Acceptance Sampling

Your factory is ordering a large number of widgets from a supplier. Each widget can be either functional or defective. The supplier guarantees that at most 3% of the widgets are defective. Since the widgets are cheap, you are actually willing to accept a rate of defectives as high as 8% before there is cause for concern.

The widgets are shipped in batches of $N = 10,000$ items. A sample of size $n = 100$ be taken from each batch to ensure that not too many widgets are defective. You and the supplier will agree on a *defective number* d so that

- If there are at least d defectives in the sample, the supplier agrees that the defective rate is greater than 3% and the batch can be rejected;
- If there are fewer than d defectives, you (the buyer) accepts the batch.

In the following, state all assumptions and/or approximations that you are making.

i) Set up a Neyman-Pearson test to decide between accepting and rejecting a batch.
(1 Mark)

ii) How large does d need to be so that any shipment containing not more than 3% defectives has at most a 1% chance of being rejected?
(2 Marks)

iii) Given this value of d, what is the probability that you end up accepting a batch with more than 8% of defectives?
(2 Marks)

Exercise 8.4 Comparing Means – Variances Known

The burning rates of two different solid-fuel propellants used in aircrew escape systems are being studied. It is known that both propellants have approximately the same standard deviation of burning rate; that is $\sigma_1 = \sigma_2 = 3$ centimeters per second. Two random samples of $n_1 = 20$ and $n_2 = 20$ specimens are tested; the sample mean burning rates are $\bar{x}_1 = 18$ centimeters per second and $\bar{x}_2 = 24$ centimeters per second.

i) Construct a 95% confidence interval on the difference in means $\mu_1 - \mu_2$. What is the practical meaning of this interval?
(2 Marks)

ii) Use the data above to decide between the hypotheses

$$H_0\colon \mu_1 = \mu_2, \qquad\qquad H_1\colon |\mu_1 - \mu_2| \geq 2.5\,\text{cm}\,.$$

Use $\alpha = 5\%$.
(2 Marks)

iii) Assuming equal sample sizes, what sample size is needed to obtain a power of 0.9 at a true difference in means of 14 cm/s?
(2 Marks)

Exercise 8.5 Comparing Means – Variances Equal

A polymer is manufactured in a batch chemical process. Viscosity measurements are normally made on each batch, and long experience with the process has indicated that the variability in the process is fairly stable. The chemical process uses a catalyst, Catalyst A, which is intended to be replaced by the more environmentally friendly Catalyst B if the viscosity is not markedly influenced by the change in catalyst.

Let μ_A denote the mean viscosity of the polymer using Catalyst A and μ_B be the mean viscosity using Catalyst B. It is hoped that the change in catalyst will not influence the mean viscosity, but if it turns out to do so significantly, then Catalyst A will not be replaced. The standard deviation of the viscosity is denoted by σ and may be assumed to be unaffected by the catalyst.

i) Roughly 95% of the time, Catalyst A will lead to a polymer viscosity in the range $\mu_A \pm 2\sigma$. If μ_B differs from μ_A by at most σ, what percentage of the polymer viscosity will at most fall outside of the range $\mu_A \pm 2\sigma$?
(2 Marks)

This percentage determined in i) is considered acceptable and catalyst A will be replaced if Catalyst B changes the mean viscosity of the polymer by less than 1 standard deviation.

ii) Formulate an appropriate hypothesis test to decide whether Catalyst A should be replaced.
(1 Mark)

iii) Given sample sizes $n_A = n_B = 20$ for the viscosities using Catalysts A and B, respectively, find the power of the test.
(2 Marks)

Pilot data yield the following viscosities:

Catalyst A	708, 732, 731, 677, 748, 702, 696, 692, 716, 729, 697, 681, 704, 740, 710, 687, 731, 704, 702, 698
Catalyst B	731, 756, 756, 762, 724, 739, 745, 715, 735, 746, 748, 770, 760, 761, 791, 720, 749, 735, 766, 738

iv) Given the above data, is the null hypothesis rejected at $\alpha = 1\%$? Quote all relevant statistics and critical values.
(3 Marks)

v) Find a 99% confidence interval on the difference in mean batch viscosity resulting from the process change.
(1 Mark)

vi) What is your conclusion regarding the catalyst change? Comment on the results of v) and vi). above. How likely is it that you have reached the wrong conclusion?
(3 Marks)