

# Ve401 Probabilistic Methods in Engineering

## Summer 2021 — Assignment 9

Date Due: 12:10 PM, Wednesday, the 21<sup>st</sup> of July 2021



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This assignment has a total of (28 Marks).

### Exercise 9.1 Comparing Means – Pooled Test

Polychlorinated Biphenyls (PCB) are worldwide environmental contaminants of industrial origin that are related to DDT. They are being phased out in the United States but they will remain in the environment for many years.

An experiment is being run to study the effects of PCB on the reproductive ability of screech owls. The purpose is to compare the shell thickness of eggs produced by birds exposed to PCB to that of birds not exposed to the contaminant. It is thought that shells of the former group will be thinner than those of the latter.

- i) Write out the null hypothesis that you will be testing.  
(1 Mark)
- ii) Do you think that Student's or Welch's T Test, or the Wilcoxon Rank-Sum Test, or none of these, or several, are appropriate here? Explain!  
(4 Marks)

The following data are obtained:

	Shell thickness, mm								
Exposed to PCB	0.21	0.223	0.25	0.19	0.20	0.226	0.215	0.24	0.136
Free of PCB	0.22	0.265	0.217	0.256	0.20	0.27	0.18	0.187	0.23

- iii) Perform a Fisher test of the Null hypothesis. What is your conclusion?  
(3 Marks)

### Exercise 9.2 Comparing Means – Paired Tests

A study of visual and auditory reaction times is conducted for a group of college basketball players. Visual reaction time is measured by the time needed to respond to a light signal, and auditory reaction time is measured by the time needed to respond to the sound of an electric switch. Fifteen subjects were measured with time recorded to the nearest millisecond:

Subject	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Visual	161	203	235	176	201	188	228	211	191	178	159	227	193	192	212
Auditory	157	207	198	161	234	197	180	165	202	193	173	137	182	159	156

Is there evidence that the visual reaction time tends to be slower than the auditory reaction time? Use a paired  $T$ -test as well as a Wilcoxon signed rank test.

(6 Marks)

### Exercise 9.3 Poisson Distribution

In 1898, Ladislaus von Bortkiewicz published the book “Das Gesetz der Kleinen Zahlen” [The Law of Small Numbers]<sup>1</sup> which detailed the use of the Poisson distribution for events that occur at a constant rate (previously, the Poisson distribution had only been used to approximate the binomial distribution).

As one of several examples, he gave data on the number annual deaths by kicks from horses of Prussian cavalrymen in 10 cavalry regiments between 1875 and 1894. In each year, the number of deaths in each regiment were reported, yielding a total of 200 reports:

No. of deaths	0	1	2	3	4
Observed Frequency	109	65	22	3	1

Perform a chi-squared goodness-of-fit test to the Poisson distribution for this data. What is the  $P$ -value of the test? What is your conclusion?

(3 Marks)

<sup>1</sup>Scanned file available at <https://archive.org/details/dasgesetzderklei00bortrich>

### Exercise 9.4 Midterm Exam Results

You are encouraged to use Mathematica as a tool for plotting, performing calculations and finding  $P$ -values in this exercise. However, the abstract derivations, degrees of freedom etc. should still be documented by hand.

The following data represents the point scores in the Midterm Exam of Ve401 in Spring 2021 (30 total points).

6.5	7.5	7.5	7.5	8	8	8.5	9.5	9.5	9.5	10
10.5	10.5	10.5	11	11	11	11.5	11.5	11.5	12	12
12	12.5	12.5	12.5	12.5	13	13	13	13	13	13.5
13.5	14	14	14	14	14	14	14	14.5	14.5	14.5
14.5	14.5	14.5	14.5	14.5	14.5	14.5	15	15	15	15
15	15.5	15.5	15.5	16	16	16	16	16.5	16.5	16.5
16.5	16.5	16.5	17	17	17	17	17	17	17	17
17	17	17	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5
17.5	17.5	18	18	18	18	18	18.5	18.5	18.5	18.5
18.5	18.5	18.5	18.5	19.5	19	19	19	19	19	19
19.5	19.5	19.5	19.5	20	20	20	20	20	20	20
20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	21	21	21
21	21	21	21	21.5	21.5	21.5	21.5	21.5	22	22
22	22	22	22	22	22.5	22.5	23.5	23.5	23.5	24
24	24	24	24	24	24.5	24.5	24.5	25	26	26.5

- Verify using a boxplot and a histogram (Freedman-Diaconis) that the data appears to be broadly consistent with a normal distribution.  
(2 Marks)
- Calculate the sample mean and sample variance of the data.  
(2 Marks)
- Perform a chi-squared goodness-of-fit test to the normal distribution. What is the  $P$ -value of the test? What is your conclusion?  
(4 Marks)

### Exercise 9.5 Independence

Does the age of a car driver influence the propensity for traffic accidents? A certain study has yielded the following data in a given year:

Age	No. of Accidents			
	0	1	2	$\geq 3$
18 – 29	748	74	31	9
30 – 39	821	60	25	10
40 – 49	786	51	22	6
50 – 59	720	66	16	5
$\geq 60$	672	50	15	7

Would you conclude that there is evidence that the age of the driver influences the number of accidents?  
(3 Marks)